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http://tinyurl.com/qo2019

Quantum Optics
Winter semester 2018/2019 - Exercise sheet 10
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Problem 1: Simplified photon counting theory.

Assume that the probability for a photon to induce a detector response (e.g. photoelectron ejection) over a certain period of time T is given by η.
a) Find the probability $P_m^{(n)}$ for the detection of m photons from a Fock state $|n\rangle$ during the interval T.
b) Considering the expansion of the density matrix of a general state in terms of Fock states and the probabilities for this state to bear given numbers of photons, find the probability P_m for the detection of m photons from a general state during the time T.
c) Show that $P_m = \rho_{mm}$ (the density matrix element of the chosen general state) if $\eta = 1$.

Problem 2: Cat states - phase distribution and phase space representations.

Consider the superposition state

$$|\alpha_+\rangle = N(|\alpha\rangle + |-\alpha\rangle),$$

where N is a normalization constant.
a) Obtain the continuous phase distribution for this state.
b) Obtain the Husimi function for this state and plot it.
c) Using the characteristic function $\chi(\lambda) = \mathrm{tr}\{\hat{\rho}\hat{D}(\lambda)\}$, where $\hat{\rho}$ is the density operator for this state and $\hat{D}(\lambda)$ is the displacement operator, obtain the Wigner function through convolution, $W(\beta) = (1/\pi^2)\int d^2\lambda \exp(\lambda^*\beta - \lambda\beta^*)\chi(\lambda)$, and plot it using an appropriate software (Wolfram Mathematica or similar). Is this a classical state?

Problem 3: Photon counting statistics.

Compute the second order correlation function $\left[g^{(2)}(\tau)\right]$ for a 2-mode Fock state $|n_{\omega_1}, n_{\omega_2}\rangle$, where $\omega_1 \neq \omega_2$ and $\mathbf{k}_1 \| \mathbf{k}_2$. Show that when $n_{\omega_1} = n_{\omega_2} = \frac{1}{2}n$, one gets

$$g^{(2)}(\tau) - 1 = \frac{1}{2}\cos(\omega_1 - \omega_2)\tau - \frac{1}{n}.$$

From there study the photon counting statistics of this state in a time interval T using

$$\mathrm{Var}\,\hat{N} - \langle\hat{N}\rangle = \frac{\langle\hat{N}\rangle^2}{T^2}\int_{-T}^{+T} \mathrm{d}\tau\ (T - |\tau|)\left[g^{(2)}(\tau) - 1\right].$$

How does it compare to the Poissonian photon counting statistics in dependence on the value of T? Are the photons bunched or anti-bunched? HINT: consider that the field amplitudes are equal.